

Isospin symmetry breaking and direct CP violation in $B \rightarrow \rho\gamma$ within and beyond the standard model *

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We discuss the isospin symmetry breaking quantity $\Delta^{\pm 0} \equiv \Gamma(B^{\pm} \rightarrow \rho^{\pm}\gamma)/2\Gamma(B^0 \rightarrow \rho^0\gamma) - 1$, and the direct CP violation in $B^{\pm} \rightarrow \rho^{\pm}\gamma$ within and beyond the standard model. After showing that the leading-order calculation would be a good approximation in such models, we argue that measurements of these quantities would be able to disentangle physics beyond the standard model.

1. INTRODUCTION

It is highly expected that the radiative decays $B \rightarrow \rho(\omega)\gamma$ would provide independent information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix, i.e. the parameters ρ and η in the Wolfenstein parametrization. This expectation is motivated by successful measurements of the radiative decays $B \rightarrow K^*\gamma$ [2] and its inclusive mode $B \rightarrow X_s\gamma$ [3]. A theoretically robust way to achieve this is by considering the ratio of $B \rightarrow X_d\gamma$ and $B \rightarrow X_s\gamma$ decay rates, which is essentially proportional to the CKM ratio $|V_{td}/V_{ts}|^2$. Hence once $B \rightarrow X_d\gamma$ is measured, this ratio would provide a measurement of $|V_{td}/V_{ts}|$ in an independent manner. We recall that the branching ratios for $B \rightarrow X_s\gamma$ [4] and $B \rightarrow X_d\gamma$ [5] in the standard model (SM) are known in next-to-leading-order (NLO).

However, as it is a challenge to measure $B \rightarrow X_d\gamma$, experimentally the exclusive decays $B \rightarrow \rho(\omega)\gamma$ are more favored and would be available much earlier, as suggested by the present experimental limits on $B \rightarrow \rho(\omega)\gamma$ [2]. As pointed long time ago, in the charged decays $B^{\pm} \rightarrow \rho^{\pm}\gamma$, the W^- annihilation [6] and also long-distance (LD) effects [7] could be significant and contribute $\sim 20\%$ to the total rate. Actually these contributions induce violation of isospin symme-

try, defined as

$$\Delta^{\pm 0} \equiv \frac{\Gamma(B^{\pm} \rightarrow \rho^{\pm}\gamma)}{2\Gamma(B^0 \rightarrow \rho^0\gamma)} - 1. \quad (1)$$

Of great interest is also direct CP asymmetry,

$$\mathcal{A}_{\text{CP}} \equiv \frac{\mathcal{B}(B^- \rightarrow \rho^- \gamma) - \mathcal{B}(B^+ \rightarrow \rho^+ \gamma)}{\mathcal{B}(B^- \rightarrow \rho^- \gamma) + \mathcal{B}(B^+ \rightarrow \rho^+ \gamma)}. \quad (2)$$

We are going to show that the quantities $\Delta^{\pm 0}$ and \mathcal{A}_{CP} are good probes to search for physics beyond the standard model.

2. THE RADIATIVE DECAYS $B \rightarrow \rho\gamma$

Now, let us concentrate on the radiative exclusive decays $B \rightarrow \rho\gamma$. The processes are governed by the following amplitude

$$\mathcal{M} = \langle \rho(p_\rho) | \mathcal{H}_{\text{eff}} | B(p_B) \rangle, \quad (3)$$

where the effective Hamiltonian describes the radiative weak-transition $b \rightarrow d\gamma$

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{G_F}{\sqrt{2}} \left[\lambda_u^{(d)} (\mathcal{C}_1(\mu) \mathcal{O}_1(\mu) + \mathcal{C}_2(\mu) \mathcal{O}_2(\mu)) \right. \\ & \left. - \lambda_d^{(t)} \mathcal{C}_7^{eff}(\mu) \mathcal{O}_7(\mu) + \dots \right]. \end{aligned} \quad (4)$$

Here, $\lambda_q^{(q')} = V_{qb} V_{qq'}^*$ are the CKM factors, and we have restricted ourselves to those contributions which will be important in what follows. The operators $\mathcal{O}_1(\mu)$ and $\mathcal{O}_2(\mu)$ are the four-quark operators

$$\mathcal{O}_1 = (\bar{d}_\alpha \Gamma^\mu u_\beta) (\bar{u}_\beta \Gamma_\mu b_\alpha), \quad (5)$$

*Based on the work in [1].

$$\mathcal{O}_2 = (\bar{d}_\alpha \Gamma^\mu u_\alpha)(\bar{u}_\beta \Gamma_\mu b_\beta), \quad (6)$$

$$\mathcal{O}_7 = \frac{em_b}{8\pi^2} \bar{d} \sigma^{\mu\nu} (1 - \gamma_5) F_{\mu\nu} b, \quad (7)$$

where $\Gamma_\mu = \gamma_\mu(1 - \gamma_5)$, α and β are the SU(3) color indices, \mathcal{C}_1 and \mathcal{C}_2 are the corresponding Wilson coefficients, and $F_{\mu\nu}$ is the electromagnetic field strength tensor.

Using the parametrisation of the form-factor, one obtains the decay width for the charged and neutral decays as

$$\begin{aligned} \Gamma^\pm &= \frac{G_F^2 \alpha |\lambda_t^{(d)}|^2}{32\pi^4} m_b^2 m_B^3 \left(1 - \frac{m_\rho^2}{m_B^2}\right)^3 |T_1^\rho|^2 \\ &\quad \times \left|1 - \frac{|\lambda_u^{(d)}|}{|\lambda_t^{(d)}|} \epsilon_A e^{i(\phi_A \mp \alpha)}\right|^2, \quad (8) \\ \Gamma^0 &= \frac{G_F^2 \alpha |\lambda_t^{(d)}|^2}{32\pi^4} m_b^2 m_B^3 \left(1 - \frac{m_\rho^2}{m_B^2}\right)^3 |T_1^\rho|^2 \quad (9) \end{aligned}$$

Here, T_1^ρ is the form-factor for the magnetic moment operator (\mathcal{C}_7) in the $B \rightarrow \rho$ transition with an on-shell photon emission. On the other hand, the second term in Eq. (8) denotes the dominant W -annihilation and the possible sub-dominant LD contributions. Keeping only the dominant W -annihilation, it reads [6]

$$\epsilon_A e^{i\phi_A} \equiv \frac{4\pi^2 m_\rho}{m_b} \frac{\mathcal{C}_2 + \mathcal{C}_1/N_c}{\mathcal{C}_7^{eff}} r_u^\rho, \quad (10)$$

where r_u^ρ is the ratio of the SD and LD form-factors induced from the penguin and the W -annihilation diagrams [6] respectively, while ϕ_A parametrises the possible strong phase induced there.

In fact, the effective Wilson coefficient \mathcal{C}_7^{eff} and the matrix elements (vertex and gluon bremsstrahlung) have been calculated up to NLO [4]. Also taking into account the relation between b quark pole mass and \overline{MS} mass up to NLO accuracy, Eq. (8) becomes

$$\begin{aligned} \Gamma^\pm &= \frac{G_F^2 \alpha |\lambda_t^{(d)}|^2}{32\pi^4} m_b^5 \left(1 - \frac{m_\rho^2}{m_B^2}\right)^3 |T_1^V|^2 \\ &\quad \times \left\{ \left| \mathcal{C}_7^{(0)eff} + A_R^{(1)t} \right|^2 \right. \\ &\quad \left. + (F_1^2 + F_2^2) \left(|A_R^u + L_R^u|^2 \right) \right. \end{aligned}$$

$$\begin{aligned} &+ 2F_1 \left[\mathcal{C}_7^{(0)eff} (A_R^u + L_R^u) + A_R^{(1)t} L_R^u \right] \\ &\left. \mp 2F_2 \left[\mathcal{C}_7^{(0)eff} A_I^u - A_I^{(1)t} L_R^u \right] \right\}, \quad (11) \end{aligned}$$

where $F_1 = -|\lambda_u^{(d)}/\lambda_t^{(d)}| \cos \alpha$ and $F_2 = -|\lambda_u^{(d)}/\lambda_t^{(d)}| \sin \alpha$, and α is one of the angles of the CKM unitarity triangle. The analogue expression for Eq. (9) can be obtained by putting $F_1 = F_2 = 0$. While $L_R^u = \epsilon_A \mathcal{C}_7^{(0)eff}$, $A_{R,I}^{(1)t}$ and $A_{R,I}^u$ are functions of the real and imaginary parts of effective Wilson coefficients and explicit $O(\alpha_s)$ contributions to the matrix elements evaluated at a scale μ ,

$$\begin{aligned} A^{(1)t} &= \frac{\alpha_s(m_b)}{4\pi} \left\{ C_7^{(1)}(\mu) - \frac{16}{3} C_7^{(0)eff}(\mu) \right. \\ &\quad \left. + \sum_i^8 C_i^{(0)eff}(\mu) \left[\gamma_{i7}^{(0)} \ln \frac{m_b}{\mu} + r_i(z) \right] \right\}, \quad (12) \end{aligned}$$

$$A^{(1)u} = \frac{\alpha_s(m_b)}{4\pi} C_2^{(0)}(\mu) [r_2(z) - r_2(0)], \quad (13)$$

where r_i 's are complex numbers and $z = (m_c/m_b)^2$.

3. ISOSPIN SYMMETRY BREAKING AND DIRECT CP VIOLATION

According to the definition given in Eq. (1), it might be better to further define the charged-conjugated averaged ratio as $\Delta \equiv \frac{1}{2} [\Delta^{-0} + \Delta^{+0}]$ that would experimentally be more accessible. It has the following expression,

$$\begin{aligned} \Delta &= 2\epsilon_A \left\{ F_1 + \frac{1}{2} \epsilon_A (F_1^2 + F_2^2) \right. \\ &\quad \left. - \frac{1}{\mathcal{C}_7^{(0)eff}} \left[F_1 A_R^{(1)t} - (F_2^2 - F_1^2) A_R^u \right] \right. \\ &\quad \left. + \epsilon_A (F_1^2 + F_2^2) (A_R^{(1)t} + F_1 A_R^u) \right\}. \quad (14) \end{aligned}$$

The first line is the leading-order (LO) expression, while the rest after it is the NLO corrections.

For the CP asymmetry, using Eqs. (2) and (11), one obtains

$$\mathcal{A}_{CP} = -\frac{2F_2}{\mathcal{C}_7^{(0)eff}(1 + \Delta_{LO})} [A_I^u - \epsilon_A A_I^t], \quad (15)$$

where Δ_{LO} is the LO part in Eq. (14). Of course, there would be additional source for the CP asym-

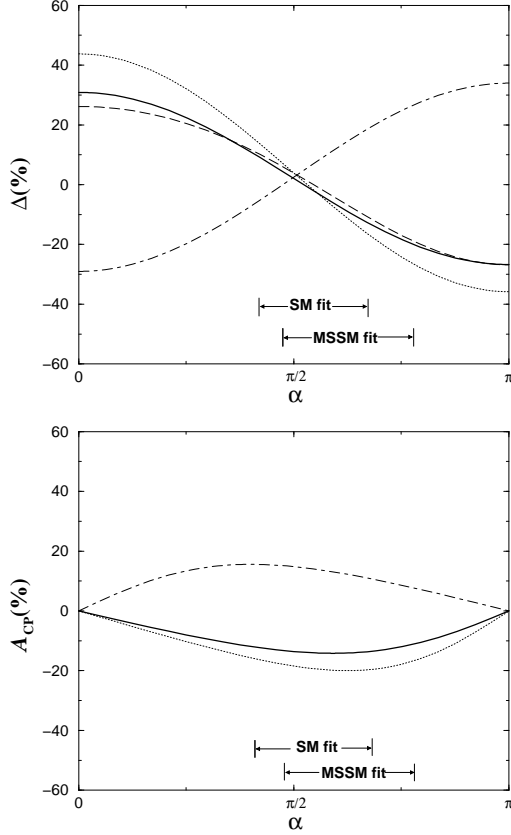


Figure 1. Δ (upper) and A_{CP} (lower) in the SM with (solid lines) and without (dashed lines) NLO corrections, and in the MSSM with large (dot-dashed lines) and small (dotted lines) $\tan\beta$.

metry if the strong phase ϕ_A is non-zero. Conversely, as long as $\phi_A = 0$, the CP asymmetry arises first at NLO where the strong phase is generated by higher order perturbative corrections.

According to Eqs. (11) and (15), Δ is essentially proportional to F_1 , while A_{CP} is proportional to F_2 . Therefore, these quantities would provide complementary measurements to determine the angle α .

4. RESULTS AND CONCLUSION

First of all, let us stress that the function $A^{(1)u}$ is universal for all models which forbid the tree-level flavor-changing-neutral-current interaction, as $A^{(1)u}$ is generated by the $u\bar{u}$ loop through the charge current interactions denoted by the operators \mathcal{O}_1 and \mathcal{O}_2 . This criterion is satisfied by the SM and in most variants of the supersymmetric (SUSY) model. Since we are going to consider only such models which satisfy this criterion, we can take the same values as the SM for A_I^t and A_I^u , i.e. $A_I^t = -0.016$ and $A_I^u = +0.046$.

For the SUSY model, we take the minimal SUSY standard model (MSSM) as a particular example. This model is appropriate to show two extreme cases where the magnitude of $\mathcal{C}_7^{(0)eff}$ could be close to the SM prediction, but its sign is either the same (in the small $\tan\beta$ region), or opposite (in the large $\tan\beta$ region) as $\mathcal{C}_7^{(0)eff(SM)}$. In the numerical analysis, we use the following values for the parameters: $(\tan\beta, \mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff(SM)}) = (3, 0.95)$ for small $\tan\beta$ region and $(\tan\beta, \mathcal{C}_7^{(0)eff}/\mathcal{C}_7^{(0)eff(SM)}) = (30, -1.2)$ for large $\tan\beta$ region [8]. The central value for the magnitude of CKM ratio in the SM is $|V_{ub}/V_{td}| = 0.48$, while in the MSSM $|V_{ub}/V_{td}| = 0.63$ which corresponds to $f = 0.6$ in [9] represents the maximum allowed contributions to $\Delta M_{d,s}$ and $|\epsilon_K|$, with $\Delta M_{d,s} = \Delta M_{d,s}^{SM}(1 + f)$. Adopting the common knowledge for the W -annihilation, we take $\epsilon_A = -0.3$.

The results are shown in Fig. (1). The SM and MSSM fits represent the allowed range of α (at 95% C.L.) from fits to the unitarity triangle in each model [9]. We should emphasize that, by definition, the angle α and $F_{1,2}$ are correlated with each other and one should keep track the uncertainties in the $\alpha - F_{1,2}$ correlation. Details can be found in the original work [1].

From the figures, it is clear that the NLO contributions in Δ are small, and then one can take into account the LO contributions as a good approximation. Although the non-zero CP asymmetry is induced at NLO accuracy, it requires only the LO $\mathcal{C}_7^{(0)eff}$ as shown explicitly in Eq. (15). While the NLO corrections would enter through

the sub-leading terms which are suppressed by ϵ_A .

Since both quantities are essentially proportional to the inverse of $\mathcal{C}_7^{(0)eff}$, they are sensitive to the sign of $\mathcal{C}_7^{(0)eff}$. For instance, if the large $\tan\beta$ MSSM solution is realized in nature, then the measured values of Δ and \mathcal{A}_{CP} could be markedly different than in the SM. These would be striking signatures of new physics and strongly suggest the presence of supersymmetry.

It should be stressed here that we have ignored all contributions where the photon and gluon lines are attached to the spectator line in $B \rightarrow \rho$ transitions, i.e. the hard spectator interactions. However, such corrections from the dropped diagrams are small as already indicated in [10]. This point will be discussed in much detail in a subsequent work [11].

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